

# Generation of strong magnetic fields in dense quark matter driven by the electroweak interaction of quarks

Maxim Dvornikov<sup>a,b,c\*</sup>

<sup>a</sup> Pushkov Institute of Terrestrial Magnetism, Ionosphere  
and Radiowave Propagation (IZMIRAN),  
108840 Troitsk, Moscow, Russia;

<sup>b</sup> Physics Faculty, National Research Tomsk State University,  
36 Lenin Avenue, 634050 Tomsk, Russia

<sup>c</sup> II. Institute for Theoretical Physics, University of Hamburg,  
149 Luruper Chaussee, D-22761 Hamburg, Germany

## Abstract

We study the generation of strong large scale magnetic fields in dense quark matter. The magnetic field growth is owing to the magnetic field instability driven by the electroweak interaction of quarks. We discuss the situation when the chiral symmetry is unbroken in the degenerate quark matter. In this case we predict the amplification of the seed magnetic field  $10^{12}$  G to the strengths  $(10^{14} - 10^{15})$  G. In our analysis we use the typical parameters of the quark matter in the core of a hybrid star or in a quark star. We also discuss the application of the obtained results to describe the magnetic fields generation in magnetars.

## 1 Introduction

The origin of strong magnetic fields  $B \sim 10^{15}$  G in some compact stars, called magnetars [1], remains an open problem of modern astrophysics. Despite the popularity of some models describing the generation of such magnetic fields, which are based on magnetohydrodynamics of stellar plasmas, none of them can satisfactory describe the observational data. These models are reviewed in Ref. [1].

Recently, the methods of elementary particle physics, mainly the chiral magnetic effect (CME) [2], were applied in Ref. [3] to generate toroidal magnetic fields in a neutron star (NS), and in particular to solve the problem of magnetars [4]. The major motivation to apply CME to produce magnetic fields in NS is that the nonzero chiral imbalance of electrons  $\mu_5 = (\mu_R - \mu_L)/2$  is created in nonequilibrium Urca processes, which are parity violating. It happens since ultrarelativistic left electrons are washed out from the system producing  $\mu_5 > 0$ . The nonzero  $\mu_5$  generates the electric current of ultrarelativistic electrons along the magnetic field. This current, in its turn, leads to the magnetic field instability resulting in the growth of a magnetic field.

---

\*maxdvo@izmiran.ru

Another possibility to utilize the electroweak interaction for the production of the magnetic field instability was proposed in Refs. [5,6]. It consists in the fact that the induced anomalous electric current along the magnetic field gets the contribution proportional to the difference of the effective potentials of the effective electroweak interaction of left and right electrons with background matter. Thus the electroweak interaction becomes a constant driver of the magnetic field instability. Then in Refs. [7–10] this idea was applied to generate strong large scale magnetic fields in NS due to the electroweak electron-nucleon interaction.

The key issue in the application of CME to generate a stellar magnetic field is the presence of left and right charged fermions in a star. Strictly speaking, the possibility to separate a fermionic field into left and right chiral projections is only possible if this particle is massless, i.e. when the chiral symmetry is unbroken. Despite the typical energy of an electron in the NS matter is much greater than its mass, one cannot claim these electrons are chiral particles there. Thus we can expect that CME for electrons is unlikely to appear in NS. This claim is also true with respect to the model in Refs. [7–10]. Note that, for the first time the fact that a nonzero particle mass destroys CME was noticed in Ref. [11]. Recently, in Ref. [12], this result of Ref. [11] was confirmed in the presence of the electroweak interaction.

Despite of the above disappointing observation, we can still expect the existence of astrophysical media where the chiral symmetry is unbroken. It is the quark matter in the core of a hybrid star (HS) or in a hypothetical quark star (QS). HS is a NS having the quark core. QS is based on the strange matter hypothesis. The properties of these compact stars are reviewed in Ref. [13]. Note that, despite of the sporadic claims of the observations of HS/QS (see, e.g., Ref. [14]), there is a certain skepticism on the existence of these compact stars.

The present work is devoted to the application of the methods of Refs. [7–10] to describe the magnetic field instability, leading to its growth, in quark matter in HS/QS. In Sec. 2, we derive the kinetic equations describing the evolution of the magnetic field and chiral imbalances in quark matter. We also formulate the initial conditions corresponding to a typical astrophysical medium. Then, in Sec. 3, we present the results of the numeric solutions of these kinetic equations. Finally, in Sec. 4, we discuss the obtained results and their applicability for modeling magnetic fields in magnetars. The computation of the helicity flip rates of quarks in their mutual collisions is provided in Appendix A.

## 2 Basic equations for the magnetic field evolution in quark matter

In this section we shall derive the equations for the evolution of the spectra of the magnetic helicity density and the magnetic energy density as well chiral imbalances in degenerate matter containing  $u$  and  $d$  quarks interacting by the parity violating electroweak forces.

Let us consider a dense quark matter consisting of  $u$  and  $d$  quarks. The density of this matter is supposed to be high enough for the chiral symmetry to be restored. In this case we can take that the quarks are effectively massless. Recently, in Ref. [15], it was shown with help of lattice simulations that the chiral symmetry has a tendency to restore in a quark matter at high density. Therefore we can decompose the quark wave functions into left and right chiral components, which evolve independently, and attribute different chemical potentials  $\mu_{qL,R}$ , where  $q = u, d$ , for each chiral component.

Generalizing the results of Refs. [7,8], we get that, in the external magnetic field  $\mathbf{B}$ , there

is the induced electric current

$$\mathbf{J} = \Pi \mathbf{B}, \quad \Pi = \frac{1}{2\pi^2} \sum_{q=u,d} e_q^2 (\mu_{q5} + V_{q5}), \quad (1)$$

where  $e_u = 2e/3$  and  $e_d = -e/3$  are the electric charges of quarks,  $e > 0$  is the elementary charge,  $\mu_{5q} = (\mu_{qR} - \mu_{qL})/2$  is the chiral imbalance,  $V_{5q} = (V_{qL} - V_{qR})/2$ , and  $V_{qL,R}$  are the effective potentials of the electroweak interaction of left and right quarks with background fermions. The potentials  $V_{qL,R}$  were found in Ref. [16] on the basis of the effective Lagrangian for the  $ud$  electroweak interaction,

$$\mathcal{L}_{\text{eff}} = - \sum_{q=u,d} \bar{q} (\gamma_0^L V_{qL} + \gamma_0^R V_{qR}) q, \quad (2)$$

where

$$\begin{aligned} V_{uL} &= -\frac{G_F}{\sqrt{2}} n_d \left( 1 - \frac{8}{3}\xi + \frac{16}{9}\xi^2 - 2|V_{ud}|^2 \right), & V_{uR} &= \frac{G_F}{\sqrt{2}} n_d \left( \frac{4}{3}\xi - \frac{16}{9}\xi^2 \right), \\ V_{dL} &= -\frac{G_F}{\sqrt{2}} n_u \left( 1 - \frac{10}{3}\xi + \frac{16}{9}\xi^2 - 2|V_{ud}|^2 \right), & V_{dR} &= \frac{G_F}{\sqrt{2}} n_u \left( \frac{2}{3}\xi - \frac{16}{9}\xi^2 \right). \end{aligned} \quad (3)$$

Here  $\gamma_0^{L,R} = \gamma_0 (1 \mp \gamma^5)/2$ ,  $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ ,  $\gamma^\mu = (\gamma^0, \boldsymbol{\gamma})$  are the Dirac matrices,  $G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2}$  is the Fermi constant,  $\xi = \sin^2 \theta_W = 0.23$  is the Weinberg parameter,  $n_{u,d}$  are the number densities of  $u$  and  $d$  quarks, and  $V_{ud} = 0.97$  is the element of the Cabbibo-Kobayashi-Maskawa matrix. The matter of the star is supposed to be electrically neutral. Thus we should have  $n_u = n_0/3$  and  $n_d = 2n_0/3$ , where  $n_0 = n_u + n_d$  is the total number density of quarks in the star. Using Eq. (3) one obtains that

$$V_{5u} = \frac{G_F}{2\sqrt{2}} \frac{2n_0}{3} \left( 2|V_{ud}|^2 + \frac{4}{3}\xi - 1 \right), \quad V_{5d} = \frac{G_F}{2\sqrt{2}} \frac{n_0}{3} \left( 2|V_{ud}|^2 + \frac{8}{3}\xi - 1 \right). \quad (4)$$

Assuming that  $n_0 = 1.8 \times 10^{38} \text{ cm}^{-3}$ , we get that  $V_{5u} = 4.5 \text{ eV}$  and  $V_{5d} = 2.9 \text{ eV}$ .

Note that in Eqs. (1) and (2) we do not account for the  $uu$  and  $dd$  interactions. However as shown in Ref. [17], basing on the direct calculation of the two loops contribution to the photon polarization operator, that such contributions to the induced current in Eq. (1) are vanishing.

Using Eq. (1) and the results of Refs. [8], we can obtain the system of kinetic equations for the spectra of the density of the magnetic helicity  $h(k, t)$  and of the magnetic energy density  $\rho_B(k, t)$ , as well as the chiral imbalances  $\mu_{5u}(t)$  and  $\mu_{5d}(t)$ , in the form,

$$\begin{aligned} \frac{\partial h(k, t)}{\partial t} &= -\frac{2k^2}{\sigma_{\text{cond}}} h(k, t) + \frac{8\alpha_{\text{em}}}{\pi\sigma_{\text{cond}}} \left\{ \frac{4}{9} [\mu_{5u}(t) + V_{5u}] + \frac{1}{9} [\mu_{5d}(t) + V_{5d}] \right\} \rho_B(k, t), \\ \frac{\partial \rho_B(k, t)}{\partial t} &= -\frac{2k^2}{\sigma_{\text{cond}}} \rho_B(k, t) + \frac{2\alpha_{\text{em}}}{\pi\sigma_{\text{cond}}} \left\{ \frac{4}{9} [\mu_{5u}(t) + V_{5u}] + \frac{1}{9} [\mu_{5d}(t) + V_{5d}] \right\} k^2 h(k, t), \\ \frac{d\mu_{5u}(t)}{dt} &= \frac{2\pi\alpha_{\text{em}}}{\mu_u^2 \sigma_{\text{cond}}} \frac{4}{9} \int dk \left[ k^2 h(k, t) \right. \\ &\quad \left. - \frac{4\alpha_{\text{em}}}{\pi} \left\{ \frac{4}{9} [\mu_{5u}(t) + V_{5u}] + \frac{1}{9} [\mu_{5d}(t) + V_{5d}] \right\} \rho_B(k, t) \right] - \Gamma_u \mu_{5u}(t), \end{aligned}$$

$$\begin{aligned} \frac{d\mu_{5d}(t)}{dt} = & \frac{2\pi\alpha_{\text{em}}}{\mu_d^2\sigma_{\text{cond}}} \frac{1}{9} \int dk \left[ k^2 h(k, t) \right. \\ & \left. - \frac{4\alpha_{\text{em}}}{\pi} \left\{ \frac{4}{9} [\mu_{5u}(t) + V_{5u}] + \frac{1}{9} [\mu_{5d}(t) + V_{5d}] \right\} \rho_B(k, t) \right] - \Gamma_d \mu_{5d}(t), \end{aligned} \quad (5)$$

where  $\Gamma_{u,d}$  are the rates for the helicity flip in  $ud$  plasma,  $\alpha_{\text{em}} = e^2/4\pi = 7.3 \times 10^{-3}$  is the QED fine structure constant,  $\sigma_{\text{cond}}$  is the electric conductivity of  $ud$  quark matter, and  $\mu_{u,d} = (3\pi^2 n_{u,d})^{1/3}$  are the mean chemical potentials of  $u$  and  $d$  quarks. In the electroneutral  $ud$  plasma, we get that  $\mu_u = (1/3)^{1/3} \mu_0 = 0.69\mu_0$  and  $\mu_d = (2/3)^{1/3} \mu_0 = 0.87\mu_0$ , where  $\mu_0 = (3\pi^2 n_0)^{1/3} = 346 \text{ MeV}$ .

The functions  $h(k, t)$  and  $\rho_B(k, t)$  in Eq. (5) are related to the total magnetic helicity  $H(t)$  and the magnetic field strength by

$$H(t) = \int d^3x (\mathbf{A} \cdot \mathbf{B}) = V \int h(k, t) dk, \quad B^2(t) = 2 \int \rho_B(k, t) dk, \quad (6)$$

where  $V$  is the normalization volume. The integration in Eq. (6) is over all the range of the wave number  $k$  variation. Note that we assume the isotropic spectra in Eq. (6).

In our model for the magnetic field generation in magnetars, we suggest that background fermions are degenerate. Nevertheless there is a nonzero temperature  $T$  of the quark matter, which is much less than the chemical potentials:  $T \ll \mu_q$ . The conductivity of the degenerate quark matter was estimated in Ref. [18] as

$$\sigma_{\text{cond}} = 4.64 \times 10^{20} \left( \alpha_s \frac{T}{T_0} \right)^{-5/3} \left( \frac{\mu_0}{300 \text{ MeV}} \right)^{8/3} \text{ s}^{-1}, \quad (7)$$

where  $\alpha_s$  is the QCD fine structure constant,  $T_0 = (10^8 - 10^9) \text{ K}$  is the initial temperature corresponding to the time  $t_0 \sim 10^2 \text{ yr}$ , when the star is already in a thermal equilibrium. Using Eq. (7), we obtain that

$$\sigma_{\text{cond}} = \sigma_0 \frac{T_0^{5/3}}{T^{5/3}}, \quad \sigma_0 = 3.15 \times 10^{22} \text{ s}^{-1}, \quad (8)$$

where we assume that  $\alpha_s \sim 0.1$ . Note that  $\sigma_{\text{cond}}$  in quark matter is several orders of magnitude less than the conductivity of electrons in the nuclear matter in NS [19].

The volume density of the internal energy of degenerate background quarks is  $\varepsilon_T = \varepsilon_0 + \delta\varepsilon_T$  [9], where  $\varepsilon_0 \sim \mu_q^4$  is the temperature independent part and  $\delta\varepsilon_T = [\mu_u^2 + \mu_d^2] T^2/2$  is the temperature correction. In Ref. [9] we suggested that the growth of the magnetic field is powered by the transmission of  $\delta\varepsilon_T$  to the magnetic energy density  $\rho_B = B^2/2$ . The energy conservation law in the magnetized  $ud$  plasma reads  $d(\delta\varepsilon_T + \rho_B)/dt = 0$  [10]. Integrating this expression with the appropriate initial condition one gets

$$[\mu_u^2 + \mu_d^2] T^2 + B^2 = [\mu_u^2 + \mu_d^2] T_0^2, \quad (9)$$

where we assume that initially the thermal energy is greater than the magnetic energy, which is the case for a young pulsar. Indeed, if one starts with a seed field  $B_0 = 10^{12} \text{ G}$ , one gets that  $\rho_B(t_0) = 1.9 \times 10^{-4} \text{ MeV}^4$  and  $\delta\varepsilon_T(t_0) = 5.5 \text{ MeV}^4$ . It means that

$$T^2 = T_0^2 \left( 1 - \frac{B^2}{B_{\text{eq}}^2} \right), \quad (10)$$

where the equipartition magnetic field can be found from the following expression [9]:

$$B_{\text{eq}}^2 = [\mu_u^2 + \mu_d^2] T_0^2 = 1.23 \mu_0^2 T_0^2. \quad (11)$$

Note that Eq. (10) describes the magnetic cooling, i.e. the temperature decreasing because of the magnetic field enhancement. As we will see later, other channels of the star cooling, such as the neutrino emission [20], are negligible on the time scale of the magnetic field growth in our model. The dependence of the temperature on the magnetic field is analogous to the quenching of the parameter  $\Pi$  in Eq. (1) introduced in Ref. [9] (see also Ref. [21]).

Although we suppose that the chiral symmetry is restored in the star and quarks are effectively massless, there are induced quark masses due to the interaction with dense matter. The effective masses of  $u$  and  $d$  quarks were computed in Ref. [22],

$$m_{u,d}^2 = \frac{e_{u,d}^2}{8\pi^2} \mu_{u,d}^2. \quad (12)$$

Note that the effective quark masses in Eq. (12) should be accounted for only in quarks collisions (see Appendix A). It implies the transitions between left and right particles in their mutual collisions. The helicity flip rates  $\Gamma_{u,d}$  for each quark types are computed in Appendix A,

$$\Gamma_u = 2.98 \times 10^{-10} \mu_0 = 1.59 \times 10^{14} \text{ s}^{-1}, \quad \Gamma_d = 5.88 \times 10^{-12} \mu_0 = 3.13 \times 10^{12} \text{ s}^{-1}, \quad (13)$$

where we use Eq. (28).

Let us introduce the following dimensionless functions:

$$\mathcal{H}(\kappa, \tau) = \frac{\alpha_{\text{em}}^2}{2\mu_0^2} h(k, t), \quad \mathcal{R}(\kappa, \tau) = \frac{\alpha_{\text{em}}^2}{k_{\text{min}} \mu_0^2} \rho_B(k, t), \quad \mathcal{M}_{u,d}(\tau) = \frac{\alpha_{\text{em}}}{\pi k_{\text{min}}} \mu_{5(u,d)}(t), \quad (14)$$

where we assume  $k_{\text{min}} < k < k_{\text{max}}$ ,  $k_{\text{min}} = 1/R = 2 \times 10^{-11} \text{ eV}$ ,  $R = 10 \text{ km}$  is the star radius,  $k_{\text{max}} = 1/\Lambda_B^{(\text{min})}$ , and  $\Lambda_B^{(\text{min})}$  is the minimal scale of the magnetic field, which is a free parameter. Using the dimensionless parameters,

$$\kappa = \frac{k}{k_{\text{min}}}, \quad \tau = \frac{2k_{\text{min}}^2}{\sigma_0} t, \quad \mathcal{V}_{u,d} = \frac{\alpha_{\text{em}}}{\pi k_{\text{min}}} V_{5(u,d)}, \quad \mathcal{G}_{u,d} = \frac{\sigma_0 \Gamma_{u,d}}{2k_{\text{min}}^2}, \quad (15)$$

as well as Eqs. (8), (10), and (13), we can rewrite Eq. (5) in the form,

$$\begin{aligned} \frac{\partial \mathcal{H}(\kappa, \tau)}{\partial \tau} &= \left(1 - \frac{B^2}{B_{\text{eq}}^2}\right)^{5/6} \left[ -\kappa^2 \mathcal{H}(\kappa, \tau) + 0.22 (4[\mathcal{M}_u(\tau) + \mathcal{V}_u] + \mathcal{M}_d(\tau) + \mathcal{V}_d) \mathcal{R}(\kappa, \tau) \right], \\ \frac{\partial \mathcal{R}(\kappa, \tau)}{\partial \tau} &= \left(1 - \frac{B^2}{B_{\text{eq}}^2}\right)^{5/6} \left[ -\kappa^2 \mathcal{R}(\kappa, \tau) + 0.22 (4[\mathcal{M}_u(\tau) + \mathcal{V}_u] + \mathcal{M}_d(\tau) + \mathcal{V}_d) \kappa^2 \mathcal{H}(\kappa, \tau) \right], \\ \frac{d\mathcal{M}_u(\tau)}{d\tau} &= 1.85 \left(1 - \frac{B^2}{B_{\text{eq}}^2}\right)^{5/6} \int_1^{\kappa_{\text{max}}} d\kappa \left[ \kappa^2 \mathcal{H}(\kappa, \tau) \right. \\ &\quad \left. - 0.22 (4[\mathcal{M}_u(\tau) + \mathcal{V}_u] + \mathcal{M}_d(\tau) + \mathcal{V}_d) \mathcal{R}(\kappa, \tau) \right] - \mathcal{G}_u \mathcal{M}_u(\tau), \\ \frac{d\mathcal{M}_d(\tau)}{d\tau} &= 0.29 \left(1 - \frac{B^2}{B_{\text{eq}}^2}\right)^{5/6} \int_1^{\kappa_{\text{max}}} d\kappa \left[ \kappa^2 \mathcal{H}(\kappa, \tau) \right. \\ &\quad \left. - 0.22 (4[\mathcal{M}_u(\tau) + \mathcal{V}_u] + \mathcal{M}_d(\tau) + \mathcal{V}_d) \mathcal{R}(\kappa, \tau) \right] - \mathcal{G}_d \mathcal{M}_d(\tau), \end{aligned} \quad (16)$$

where  $\kappa_{\max} = k_{\max}/k_{\min}$ ,  $B^2$  and  $B_{\text{eq}}^2$  are given in Eqs. (6) and (11).

While solving of Eq. (16) numerically, we use the initial Kolmogorov spectrum of the magnetic energy density,  $\rho_B(k, t_0) = \mathcal{C}k^{-5/3}$ , where the constant  $\mathcal{C}$  can be obtained by equating the initial magnetic energy density, computed on the basis of Eq. (6), to  $B_0^2/2$  (see Ref. [8]). The initial spectrum of the magnetic helicity density is  $h(k, t_0) = 2r\rho_B(k, t_0)/k$ , where the parameter  $0 \leq r \leq 1$ , corresponds to initially nonhelical,  $r = 0$ , and maximally helical,  $r = 1$ , fields.

In Ref. [7] we found that the evolution of the magnetic field is almost independent on the initial values of the chiral imbalances  $\mu_{5(u,d)}(t_0)$  because of the huge helicity flip rates  $\Gamma_{u,d}$ . Therefore we can take almost arbitrary values of  $\mu_{5(u,d)}(t_0)$  only requiring that  $\mu_{5(u,d)}(t_0) \ll \mu_{u,d}$ . In our simulations we shall take that  $\mu_{5u}(t_0) = \mu_{5d}(t_0) = 1 \text{ MeV}$ .

### 3 Results of the numeric solution of the kinetic equations

In this section we present the results of the numerical solution of Eq. (16) with the initial conditions corresponding to a quark matter in a compact star.

In Fig. 1 we show the amplification of the initial magnetic field  $B_0 = 10^{12} \text{ G}$  by two or three orders of magnitude. This result is obtained by numerically solving Eq. (16) with the initial conditions discussed in Sec. 2. These initial conditions are quite possible in a dense quark matter in a HS/QS.

One can see in Fig. 1 that the magnetic field reaches the saturated strength  $B_{\text{sat}}$ . This result is analogous to the findings of Refs. [9, 10]. For  $T_0 = 10^8 \text{ K}$  in Figs. 1(a) and 1(b),  $B_{\text{sat}} \approx 1.1 \times 10^{14} \text{ G}$ ; and for  $T_0 = 10^9 \text{ K}$  in Figs. 1(c) and 1(d),  $B_{\text{sat}} \approx 1.1 \times 10^{15} \text{ G}$ . However, unlike Refs. [9, 10],  $B_{\text{sat}}$  in Fig. 1 is defined entirely by  $T_0$ . The obtained  $B_{\text{sat}}$  is close to the magnetic field strength predicted in magnetars [1], especially for  $T_0 = 10^9 \text{ K}$ .

The time of the magnetic field growth to  $B_{\text{sat}}$  is several orders of magnitude shorter than in Refs. [9, 10]. This fact is due to the smaller value of the electric conductivity  $\sigma_{\text{cond}}$  in quark matter in Eq. (7) compared to  $\sigma_{\text{cond}}$  for electrons in nuclear matter which we used in Refs. [9, 10]. This fact can be understood with help of the Faraday equation,

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{\Pi}{\sigma_{\text{cond}}} (\nabla \times \mathbf{B}) + \frac{1}{\sigma_{\text{cond}}} \nabla^2 \mathbf{B}, \quad (17)$$

which is equivalent to the first two lines in Eq. (5). Using Eq. (17) one gets that the saturation time  $t_{\text{sat}} \sim \sigma_{\text{cond}}/\Pi\Lambda_B$ , where  $\Lambda_B$  is the magnetic field scale. It means that the smaller  $\sigma_{\text{cond}}$  is, the faster the magnetic field reaches  $B_{\text{sat}}$ . Moreover, we can see that short scale magnetic field should reach  $B_{\text{sat}}$  faster. The later fact, which was also established in Refs. [8–10], is confirmed by the comparison of Figs. 1(a) and 1(b) as well as Figs. 1(c) and 1(d).

In our model of the magnetic field generation, the thermal energy of background fermions is converted to the magnetic energy. One can say that a star cools down magnetically. The typical values of  $t_{\text{sat}}$  are  $\lesssim 10 \text{ h}$  in Figs. 1(a) and 1(b) and  $\lesssim 10^2 \text{ min}$  in Figs. 1(c) and 1(d). At such short time scales, other cooling channels, such as that due to the neutrino emission [20], do not contribute to the temperature evolution significantly. Therefore, unlike Refs. [8–10], we omit them in our present simulations.

In Fig. 1 we can see that, although the initial magnetic helicity can be different (see solid and dashed lines there), the subsequent evolution of such magnetic fields is almost indistinguishable, especially at  $t \sim t_{\text{sat}}$ . It means that, besides the generation of strong

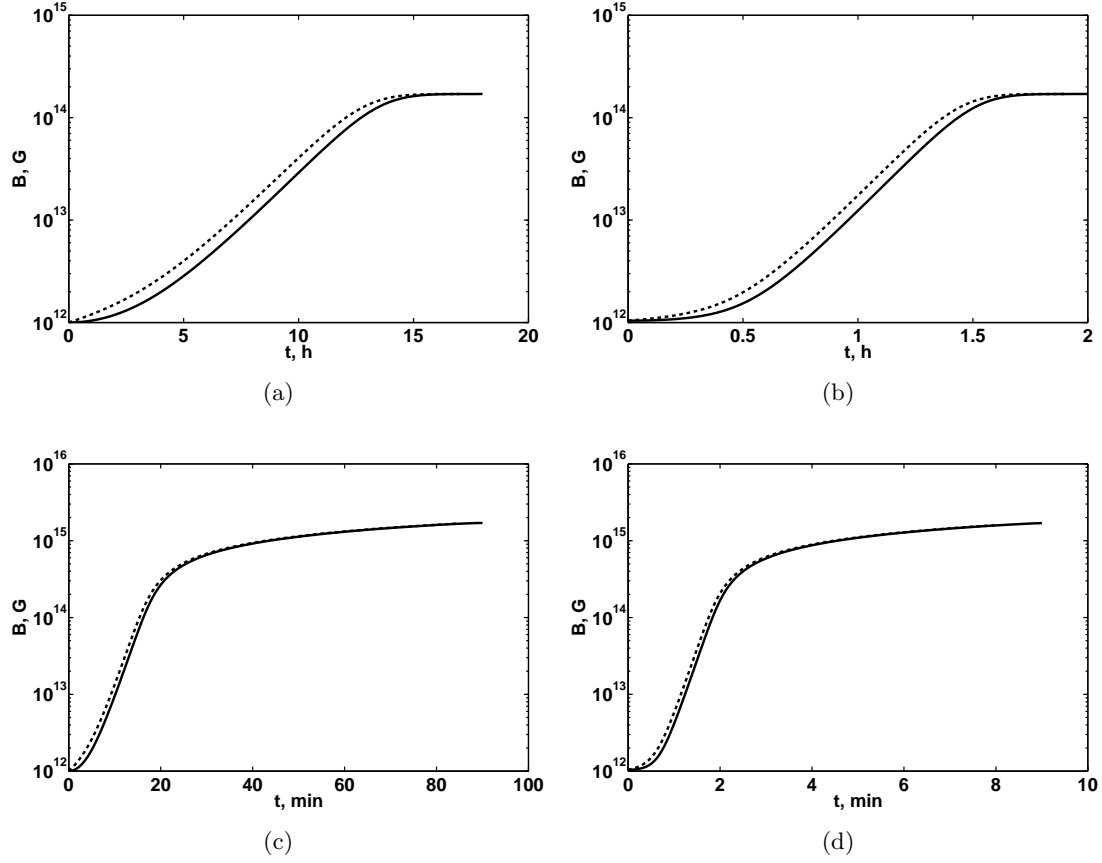


Figure 1: The magnetic field versus time for different initial temperatures  $T_0$  and minimal length scales  $\Lambda_B^{(\min)}$ . The solid lines correspond to initially nonhelical fields with  $r = 0$  and dashed ones to the fields having maximal initial helicity,  $r = 1$ . (a)  $T_0 = 10^8$  K and  $\Lambda_B^{(\min)} = 1$  km. (b)  $T_0 = 10^8$  K and  $\Lambda_B^{(\min)} = 100$  m. (c)  $T_0 = 10^9$  K and  $\Lambda_B^{(\min)} = 1$  km. (d)  $T_0 = 10^9$  K and  $\Lambda_B^{(\min)} = 100$  m.

magnetic field, we also generate the magnetic helicity in quark matter. This result is in the agreement with Refs. [8–10].

## 4 Discussion

In the present work we have applied the mechanism for the magnetic field generation, proposed in Refs. [7–9], to create strong large scale magnetic fields in dense quark matter. This mechanism is based on the magnetic field instability driven a parity violating electroweak interaction between particles in the system. We have established the system of kinetic equations for the spectra of the magnetic helicity density and the magnetic energy density, as well as for the chiral imbalances, and have solved it numerically.

Although there is a one-to-one correspondence between the mechanisms for the magnetic field generation in Refs. [7–10] and in the present work, the scenario described here is likely to be more realistic. As mentioned in Ref. [12] the generation of the anomalous current in Eq. (1) is impossible for massive particles. Electrons in NS are ultrarelativistic but have a nonzero mass. As found in Ref. [23], the chiral symmetry can be restored at densities  $n \sim M_W^3 \sim 10^{46} \text{ cm}^{-3}$ , that is much higher than one can expect in NS. Therefore the chiral magnetic effect for electrons as well as the results of Refs. [7–10] are unlikely to be applied in NS. Recently this fact was also mentioned in Ref. [12].

On the contrary, the chiral symmetry was found in Ref. [24] to be restored for lightest  $u$  and  $d$  quarks even at densities corresponding to a core of HS or in QS. Accounting for the existence of the electroweak parity violating interaction between  $u$  and  $d$  quarks, we can conclude that the application of the methods of Refs. [7–10] to the quark matter in a compact star is quite plausible.

We have obtained that, in quark matter, the seed magnetic field  $B_0 = 10^{12} \text{ G}$ , which is typical in a young pulsar, is amplified up to  $B_{\text{sat}} \sim (10^{14} - 10^{15}) \text{ G}$ , depending on the initial temperature. Such magnetic fields are predicted in magnetars [1]. Therefore HS/QS can become a magnetar. The obtained growth time of the magnetic field to  $B_{\text{sat}}$  is much less than that in electron-nucleon case studied in Refs. [7–10]. It means that, in our model, strong magnetic fields are generated quite rapidly with  $t_{\text{sat}} \sim$  several hours after a star is in a thermal equilibrium.

Note that, in the present work, instead of the quenching of the parameter  $\Pi$  in Eq. (1) suggested in Ref. [9] to avoid the excessive growth of the magnetic field, we used the conservation of the total energy in Eq. (10) and the dependence of the electric conductivity on the temperature in Eq. (8); cf. Ref. [10]. It results in a more explicit saturation of the magnetic field in Eq. (16); cf. Fig. 1.

Despite the plausibility of the results, several important assumptions were made. Firstly, while calculating the helicity flip rates in Appendix A, we have taken that quarks exchange by plasmons in their scattering. It is, however, known (see, e.g., Ref. [25]) that modified effective interaction potentials can exist in a dense degenerate matter. If one takes into account these interactions it can somehow change the values of  $\Gamma_{u,d}$ . Nevertheless, since the present work is a qualitative study of the magnetic field generation in the degenerate quark matter, we shall restrict ourselves to the plasmon interaction of quarks.

Secondly, we have considered the simplest case of a compact star consisting of only  $u$  and  $d$  quarks. However, strange stars, having a certain fraction of  $s$  quarks are also actively studied [13, pp. 414–440]. The nonzero fraction of  $s$  quarks, which cannot exceed  $1/3$ , is also



required by the beta equilibrium. Nonetheless  $s$  quarks are unlikely to contribute significantly to the generation of magnetic fields in our model. Firstly, the mass of an  $s$  quark  $m_s = 150$  MeV is quite great, i.e. the chiral symmetry will remain broken for these particles. Thus,  $s$  quarks do not contribute to the induced current in Eq. (1). Secondly, even if  $s$  quarks contribute to the helicity flip rates of  $u$  and  $d$  quarks, it will not change the evolution of the magnetic field. Indeed,  $\Gamma_{u,d}$  computed in Appendix A, is already great enough to wash out the initial chiral imbalances  $\mu_{5(u,d)}(0)$ . Any bigger contribution to  $\Gamma_{u,d}$  will eliminate  $\mu_{5(u,d)}(0)$  faster. However, the growth of the magnetic field is driven by  $V_{5(u,d)}$ , which is constant, rather than by  $\mu_{5(u,d)}$ .

Summarizing, we have described the generation of strong large scale magnetic fields in dense quark matter driven by the magnetic field instability caused by the electroweak interaction of quarks. The described phenomenon may well exist in the core of HS or in QS. We suggest that the obtained results can have implication to the problem of magnetars since the generated magnetic fields have strength close to that predicted in these highly magnetized compact stars.

## Acknowledgements

I am thankful to S.I. Blinnikov, A.V. Borisov, V.V. Braguta, M.I. Krivoruchenko, N. Leite, A.E. Lobanov, B.V. Martemyanov, K.A. Postnov, V.B. Semikoz, G. Sigl, M.I. Vysotsky, V.I. Zakharov, and V.Ch. Zhukovsky for useful discussions, as well as to the Tomsk State University Competitiveness Improvement Program, RFBR (research project No. 15-02-00293), and DAAD (grant No. 91610946) for partial support.

## A Helicity flip rates in degenerate quark matter

In this Appendix we shall compute the helicity flip rates of  $u$  and  $d$  quarks in their collisions in dense matter as well as derive the kinetic equations for the chiral imbalances.

As mentioned in Sec. 2, quarks acquire effective masses in dense matter. Thus the helicity of quarks will change when the particles collide. There are three types of reactions: (a) scattering of identical quarks, with helicities of both particles being changed; (b) scattering of different quark flavors, with helicities of both particles being changed; and (c) scattering of different quark flavors, with helicity of only one particle being changed. We shall successively discuss all the cases. Quarks are supposed to interact by the plasmon exchange.

**Scattering of identical quarks** There are four reactions in this group:  $u_L u_L \rightarrow u_R u_R$ ,  $d_L d_L \rightarrow d_R d_R$ ,  $u_R u_R \rightarrow u_L u_L$ , and  $d_R d_R \rightarrow d_L d_L$ . We study in details only the process  $u_L(p_1) + u_L(p_2) \rightarrow u_R(p'_1) + u_R(p'_2)$ , where  $p_{1,2}^\mu = (E_{1,2}, \mathbf{p}_{1,2})$  are the momenta of incoming quarks and  $p_{1,2}'^\mu = (E'_{1,2}, \mathbf{p}'_{1,2})$  are the momenta of outgoing quarks. In this reaction, the number of left particles is decreased by two units and the number of right particles is increased by two units. Other reactions in this group can be studied analogously.

The matrix element has the form,

$$\mathcal{M} = ie_u^2 \left[ \frac{1}{t} \bar{u}(p'_1) \gamma^\mu u(p_1) \cdot \bar{u}(p'_2) \gamma_\mu u(p_2) - \frac{1}{u} \bar{u}(p'_2) \gamma^\mu u(p_1) \cdot \bar{u}(p'_1) \gamma_\mu u(p_2) \right]. \quad (18)$$

where  $t = (p'_1 - p_1)^2$  and  $u = (p'_2 - p_1)^2$  are the Mandelstam variables. The square of the matrix element in Eq. (18) is

$$|\mathcal{M}|^2 = e_u^4 \left[ \frac{1}{t^2} \text{tr}(\rho'_2 \gamma_\mu \rho_2 \gamma_\nu) \cdot \text{tr}(\rho'_1 \gamma^\mu \rho_1 \gamma^\nu) + \frac{1}{u^2} \text{tr}(\rho'_1 \gamma_\mu \rho_2 \gamma_\nu) \cdot \text{tr}(\rho'_2 \gamma^\mu \rho_1 \gamma^\nu) \right. \\ \left. - \frac{1}{tu} \text{tr}(\rho'_2 \gamma_\mu \rho_2 \gamma_\nu \rho'_1 \gamma^\mu \rho_1 \gamma^\nu) - \frac{1}{tu} \text{tr}(\rho'_1 \gamma_\mu \rho_2 \gamma_\nu \rho'_2 \gamma^\mu \rho_1 \gamma^\nu) \right], \quad (19)$$

where the density matrices are [26, pp. 106–111]

$$\rho_{1,2} = \frac{1}{2} [(\gamma \cdot p_{1,2}) + m_u] [1 + \gamma^5 (\gamma \cdot a_{1,2})], \\ \rho'_{1,2} = \frac{1}{2} [(\gamma \cdot p'_{1,2}) + m_u] [1 + \gamma^5 (\gamma \cdot a'_{1,2})]. \quad (20)$$

Here  $m_u$  is the effective mass given in Eq. (12) and the polarization vectors are [10]

$$a_{1,2}^\mu = \frac{1}{m_u} (-p_{1,2}, -E_{1,2} \mathbf{n}_{1,2}), \quad a'_{1,2}^\mu = \frac{1}{m_u} (p'_{1,2}, E'_{1,2} \mathbf{n}'_{1,2}), \quad (21)$$

which correspond to left and right particles. Here  $\mathbf{n}_{1,2}$  and  $\mathbf{n}'_{1,2}$  are the unit vectors along  $\mathbf{p}_{1,2}$  and  $\mathbf{p}'_{1,2}$ .

Choosing the center-of-mass frame of colliding quarks and assuming the elastic scattering, one gets that

$$\text{tr}(\rho'_2 \gamma_\mu \rho_2 \gamma_\nu) \cdot \text{tr}(\rho'_1 \gamma^\mu \rho_1 \gamma^\nu) = 16m_u^4 \sin^4 \frac{\theta_{\text{cm}}}{2}, \\ \text{tr}(\rho'_1 \gamma_\mu \rho_2 \gamma_\nu) \cdot \text{tr}(\rho'_2 \gamma^\mu \rho_1 \gamma^\nu) = 16m_u^4 \cos^4 \frac{\theta_{\text{cm}}}{2}, \\ \text{tr}(\rho'_2 \gamma_\mu \rho_2 \gamma_\nu \rho'_1 \gamma^\mu \rho_1 \gamma^\nu) = \text{tr}(\rho'_1 \gamma_\mu \rho_2 \gamma_\nu \rho'_2 \gamma^\mu \rho_1 \gamma^\nu) = -4m_u^4 \sin^2 \theta_{\text{cm}}, \quad (22)$$

where  $\theta_{\text{cm}}$  is the scattering angle, i.e. the angle between  $\mathbf{p}_1$  and  $\mathbf{p}'_1$  in the center-of-mass frame. In the same frame one has

$$t = -2E_{\text{cm}}^2 (1 - \cos \theta_{\text{cm}}), \quad u = -2E_{\text{cm}}^2 (1 + \cos \theta_{\text{cm}}), \quad (23)$$

where  $E_{\text{cm}}$  is the energy of colliding quarks in the center-of-mass frame. In Eq. (23), we also assume that the scattering is elastic. We can express  $E_{\text{cm}}$  in term of the variables in the laboratory frame, i.e. where the star is at rest, as  $E_{\text{cm}}^2 \approx \{m_u^2 + E_1 E_2 [1 - (\mathbf{n}_1 \cdot \mathbf{n}_2)]\} / 2$ . Since we study the probability in the lowest order in the effective mass and traces in Eq. (22) are proportional to  $m_u^4$ , we neglect  $m_u$  in Eq. (23) as well as in the following calculations.

Finally, Eq. (19) takes the form

$$|\mathcal{M}|^2 = \frac{16m_u^4 e_u^4}{\{m_u^2 + E_1 E_2 [1 - (\mathbf{n}_1 \cdot \mathbf{n}_2)]\}^2}. \quad (24)$$

The total probability of the process has the form [26, pp. 247–252],

$$W = \frac{V}{64(2\pi)^8} \int \frac{d^3 p_1 d^3 p_2 d^3 p'_1 d^3 p'_2}{E_1 E_2 E'_1 E'_2} \delta^4(p_1 + p_2 - p'_1 - p'_2) |\mathcal{M}|^2 \\ \times f(E_1 - \mu_{uL}) f(E_2 - \mu_{uL}) [1 - f(E'_1 - \mu_{uR})] [1 - f(E'_2 - \mu_{uR})], \quad (25)$$

where  $f(E) = [\exp(\beta E) + 1]^{-1}$  is the Fermi-Dirac distribution of quarks,  $\beta = 1/T$  is the reciprocal temperature,  $\mu_{L,R}$  are the chemical potentials of left and right quarks, and  $|\mathcal{M}|^2$  is given in Eq. (24). Here we assume that quarks are degenerate, i.e.  $f(E - \mu) = \Theta(\mu - E)$ , where  $\Theta(z)$  is the Heaviside step function. In Eq. (25) we introduce the additional factor  $4 = 2! \times 2!$  in the denominator to take into account identical particles in the initial and final states. The direct calculation of the integrals over the phase space in Eq. (25) accounting for  $|\mathcal{M}|^2$  in Eq. (24) gives

$$W(u_L u_L \rightarrow u_R u_R) = \frac{e_u^4 m_u^2 \mu_u V}{8\pi^5} (\mu_{uL} - \mu_{uR}) \Theta(\mu_{uL} - \mu_{uR}). \quad (26)$$

Analogously we can compute the probabilities of other reactions in this group.

The kinetic equations for the evolution of the total number of left and right  $u$  quarks  $N_{uL,R}$  are

$$\begin{aligned} \frac{dN_{uL}}{dt} &= -2W(u_L u_L \rightarrow u_R u_R) + 2W(u_R u_R \rightarrow u_L u_L), \\ \frac{dN_{uR}}{dt} &= +2W(u_L u_L \rightarrow u_R u_R) - 2W(u_R u_R \rightarrow u_L u_L), \end{aligned} \quad (27)$$

Accounting for Eq. (26) and the analogous expression for  $d$  quarks, one gets the evolution of the chiral imbalances  $\mu_{5(u,d)} = (\mu_{(u,d)R} - \mu_{(u,d)L})/2$  in the form,

$$\dot{\mu}_{5(u,d)} = -\Gamma_{u,d} \mu_{5(u,d)}, \quad \Gamma_{u,d} = \frac{e_{u,d}^4 m_{u,d}^2}{\pi^3 \mu_{u,d}}. \quad (28)$$

In Eq. (28) we take into account the relation between the number densities  $n_{L,R} = N_{L,R}/V$  and chemical potentials of left and right quarks

$$n_{L,R} = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\exp[\beta(p - \mu_{L,R})] + 1} \approx \frac{\mu_{L,R}^3}{6\pi^2}, \quad (29)$$

where we assume that massless quarks have only one polarization. In particular we get from Eq. (29) that  $d(n_{(u,d)R} - n_{(u,d)L})/dt \approx \dot{\mu}_{5(u,d)} \mu_{u,d}^2 / \pi^2$ .

**Scattering of  $ud$  quarks: both particles change helicity** There are also four reactions:  $u_L d_L \rightarrow u_R d_R$ ,  $u_R d_R \rightarrow u_L d_L$ ,  $u_L d_R \rightarrow u_R d_L$ , and  $u_R d_L \rightarrow u_L d_R$  in this group. Let us first study the following process:  $u_L(p_1) + d_L(p_2) \rightarrow u_R(p'_1) + d_R(p'_2)$ . The matrix element has the form,

$$\mathcal{M} = ie_u e_d \frac{\bar{u}(p'_1) \gamma^\mu u(p_1) \cdot \bar{d}(p'_2) \gamma_\mu d(p_2)}{(p'_1 - p_1)^2}. \quad (30)$$

Instead of using Eqs. (20) and (21) to compute  $|\mathcal{M}|^2$ , we can utilize the solution of the Dirac equation, corresponding to left and right particles

$$\begin{aligned} u_L(p_1) &= \sqrt{E_1 + p_1} \begin{pmatrix} -\frac{m_u}{E_1 + p_1} w_-(\mathbf{p}_1) \\ w_-(\mathbf{p}_1) \end{pmatrix}, \\ u_R(p'_1) &= \sqrt{E'_1 + p'_1} \begin{pmatrix} w_+(\mathbf{p}'_1) \\ -\frac{m_u}{E'_1 + p'_1} w_+(\mathbf{p}'_1) \end{pmatrix} \end{aligned} \quad (31)$$

where  $w_{\pm}(\mathbf{p})$  are the helicity amplitudes which can be found in Ref. [26, p. 86]. The spinors in Eq. (31) are normalized as  $\bar{u}u = 2m_u$ . Analogous spinors are valid for  $d$  quarks. The direct calculation of  $|\mathcal{M}|^2$  with help of Eq. (31) gives

$$|\mathcal{M}|^2 = \frac{e_u^2 e_d^2 m_u^2 m_d^2}{16E_1 E_2 E'_1 E'_2} \frac{(E'_1 + p'_1 + E_1 + p_1)^2 (E'_2 + p'_2 + E_2 + p_2)^2}{(E'_1 + p'_1)(E_1 + p_1)(E'_2 + p'_2)(E_2 + p_2)}, \quad (32)$$

where we keep the leading term in the effective quark masses and assume the elastic scattering.

Analogously to Eq. (25) one obtains the total probability for the reaction  $u_L d_L \rightarrow u_R d_R$  in the form,

$$W = \frac{V}{16(2\pi)^8} \int \frac{d^3 p_1 d^3 p_2 d^3 p'_1 d^3 p'_2}{E_1 E_2 E'_1 E'_2} \delta^4(p_1 + p_2 - p'_1 - p'_2) |\mathcal{M}|^2 \times \Theta(\mu_{uL} - E_1) \Theta(\mu_{dL} - E_2) \Theta(E'_1 - \mu_{uR}) \Theta(E'_2 - \mu_{dR}). \quad (33)$$

After the computation of the integrals over the quark momenta in Eq. (33) one has

$$W(u_L d_L \rightarrow u_R d_R) = W_0 (\mu_{uL} + \mu_{dL} - \mu_{uR} - \mu_{dR}) \Theta(\mu_{uL} + \mu_{dL} - \mu_{uR} - \mu_{dR}), \quad (34)$$

where  $W_0 \sim e_u^2 e_d^2 m_u^2 m_d^2 V / \sqrt{\mu_u \mu_d}$ . Comparing  $W(u_L d_L \rightarrow u_R d_R)$  with, e.g.,  $W(u_L u_L \rightarrow u_R u_R)$  in Eq. (26), one gets that  $W(u_L d_L \rightarrow u_R d_R) \ll W(u_L u_L \rightarrow u_R u_R)$  since  $m_{u,d}^2 \sim \alpha_{\text{em}} \mu_{u,d}^2 \ll \mu_{u,d}^2$  (see Eq. (12) and Ref. [22]). It means that the contribution of the reactions in the considered group to the helicity flip rates is negligible.

**Scattering of  $ud$  quarks: only one particle changes helicity** One has eight reactions  $u_L d_{L,R} \rightarrow u_R d_{L,R}$ ,  $u_R d_{L,R} \rightarrow u_L d_{L,R}$ ,  $d_L u_{L,R} \rightarrow d_R u_{L,R}$ , and  $d_R u_{L,R} \rightarrow d_L u_{L,R}$  present in this group. Let us first study the process  $u_L(p_1) + d_L(p_2) \rightarrow u_R(p'_1) + d_L(p'_2)$ . The matrix element for this reaction is

$$\mathcal{M} = ie_u e_d \frac{\bar{u}(p'_1) \gamma^\mu u(p_1) \cdot \bar{d}(p'_2) \gamma_\mu d(p_2)}{(p'_1 - p_1)^2}. \quad (35)$$

The calculation of  $|\mathcal{M}|^2$  can be made with help of Eq. (31). Here we present only the final result,

$$|\mathcal{M}|^2 = m_u^2 e_u^2 e_d^2 \frac{E_2 E'_2}{E_1^2 E_1'^2} \frac{[1 + (\mathbf{n}_2 \cdot \mathbf{n}'_2)]}{[1 - (\mathbf{n}_1 \cdot \mathbf{n}'_1)]}, \quad (36)$$

where as usual we assume that quarks are ultrarelativistic and the scattering is elastic.

The total probability of the process  $u_L d_L \rightarrow u_R d_L$  is

$$W = \frac{V}{16(2\pi)^8} \int \frac{d^3 p_1 d^3 p_2 d^3 p'_1 d^3 p'_2}{E_1 E_2 E'_1 E'_2} \delta^4(p_1 + p_2 - p'_1 - p'_2) |\mathcal{M}|^2 \times \Theta(\mu_{uL} - E_1) \Theta(E'_1 - \mu_{uR}) f(E_2 - \mu_{dL}) [1 - f(E'_2 - \mu_{dL})], \quad (37)$$

where  $|\mathcal{M}|^2$  is given in Eq. (36). Note that, in Eq. (37) we do not replace the initial and final distributions of  $d$  quarks with step functions since  $d$  quark does not change the helicity. The integration over the particles momenta can be made as in Ref. [10]. Here we present only the final result,

$$W(u_L d_L \rightarrow u_R d_L) = \frac{e_u^2 e_d^2}{16\pi^5} V \mu_{dL}^3 \frac{T}{\omega_p} \left( \frac{m_u}{\mu_u} \right)^2 (\mu_{uL} - \mu_{uR}) \Theta(\mu_{uL} - \mu_{uR}). \quad (38)$$

We just mention that, to get Eq. (38) we have to avoid the infrared divergence. For this purpose we introduce the plasma frequency

$$\omega_p = \frac{1}{\sqrt{3}\pi} \sqrt{e_u^2 \mu_u^2 + e_d^2 \mu_d^2} = 3.04 \times 10^{-2} \mu_0, \quad (39)$$

in the degenerate  $ud$  quark matter [27]. Comparing Eq. (38) with Eq. (26) one can see that  $W(u_L d_L \rightarrow u_R d_L) \ll W(u_L u_L \rightarrow u_R u_R)$  since  $T \ll \omega_p$  in the degenerate matter. That is why the reactions in this group can be omitted as well.

At the end of this Appendix we mention that we do not study the influence of the electroweak interaction between quarks on the helicity flip in quark collisions. The contribution of the electroweak interaction to the scattering probability of electrons off protons was studied in Ref. [10], where it was found that  $V_5$  does not enter to the analog of Eq. (28) for the evolution of the chiral imbalance  $\mu_5$ .

## References

- [1] R. Turolla, S. Zane, A.L. Watts, Magnetars: the physics behind observations. A review, Rep. Prog. Phys. 78 (2015) 116901, arXiv:1507.02924.
- [2] V.A. Miransky, I.A. Shovkovy, Quantum field theory in a magnetic field: From quantum chromodynamics to graphene and Dirac semimetals, Phys. Rept. 576 1 (2015) 1–209, arXiv:1503.00732.
- [3] J. Charbonneau, A. Zhitnitsky, Topological currents in neutron stars: Kicks, precession, toroidal fields and magnetic helicity, J. Cosmol. Astropart. Phys. 08 (2010) 010, arXiv:0903.4450.
- [4] G. Sigl, N. Leite, Chiral magnetic effect in protoneutron stars and magnetic field spectral evolution, J. Cosmol. Astropart. Phys. 01 (2016) 025, arXiv:1507.04983.
- [5] A. Boyarsky, O. Ruchayskiy, M. Shaposhnikov, Long-range magnetic fields in the ground state of the Standard Model plasma, Phys. Rev. Lett. 109 (2012) 111602, arXiv:1204.3604.
- [6] M. Dvornikov, V.B. Semikoz, Instability of magnetic fields in electroweak plasma driven by neutrino asymmetries, J. Cosmol. Astropart. Phys. 05 (2015) 002, arXiv:1311.5267.
- [7] M. Dvornikov, V.B. Semikoz, Magnetic field instability in a neutron star driven by the electroweak electron-nucleon interaction versus the chiral magnetic effect, Phys. Rev. D 91 (2015) 061301, arXiv:1410.6676.
- [8] M. Dvornikov, V.B. Semikoz, Generation of the magnetic helicity in a neutron star driven by the electroweak electron-nucleon interaction, J. Cosmol. Astropart. Phys. 05 (2015) 032, arXiv:1503.04162.
- [9] M. Dvornikov, V. B. Semikoz, Energy source for the magnetic field growth in magnetars driven by the electron-nucleon interaction, Phys. Rev. D 92 (2015) 083007, arXiv:1507.03948.

- [10] M. Dvornikov, Relaxation of the chiral imbalance and the generation of magnetic fields in magnetars, to be published in J. Exp. Theor. Phys. (2016), arXiv:1510.06228.
- [11] A. Vilenkin, Equilibrium parity violating current in a magnetic field, Phys. Rev. D 22 (1980) 3080–3084.
- [12] M. Dvornikov, Role of particle masses in the magnetic field generation driven by the parity violating interaction, Phys. Lett. B 760 (2016) 406–410, arXiv:1608.04940.
- [13] N. K. Glendenning, Compact Stars: Nuclear Physics, Particle Physics, and General Relativity, 2nd ed., Springer, New York, 2000.
- [14] Z.G. Dai, S.Q. Wang, J.S. Wang, L.J. Wang, Y.W. Yu, The most luminous supernova ASASSN-15LH: Signature of a newborn rapidly rotating strange quark star, Astrophys. J. 817 (2016) 132–137, arXiv:1508.07745.
- [15] V.V. Braguta, E.-M. Ilgenfritz, A.Yu. Kotov, A.V. Molochkov, A.A. Nikolaev, Study of the phase diagram of dense two-color QCD within lattice simulation, arXiv:1605.04090.
- [16] M. Dvornikov, Galvano-rotational effect induced by electroweak interactions in pulsars, J. Cosmol. Astropart. Phys. 05 (2015) 037, arXiv:1503.00608.
- [17] M. Dvornikov, Impossibility of the strong magnetic fields generation in an electron-positron plasma, Phys. Rev. D 90 (2014) 041702, arXiv:1405.3059.
- [18] H. Heiselberg, C.J. Pethick, Transport and relaxation in degenerate quark plasmas, Phys. Rev. D 48 (1993) 2916–2928.
- [19] D.C. Kelly, Electrical and thermal conductivities of a relativistic degenerate plasma, Astrophys. J. 179 (1973) 599–606.
- [20] D.G. Yakovlev, A.D. Kaminker, O.Y. Gnedin, P. Haensel, Neutrino emission from neutron stars, Phys. Rept. 354 (2001) 1–155, astro-ph/0012122.
- [21] If we suppose that  $B^2 \ll B_{\text{eq}}^2$  in Eq. (10), we can rewrite Eq. (10) as  $T^2 \sim (1 + B^2/B_{\text{eq}}^2)^{-1}$ . Then, we account for that  $\sigma_{\text{cond}} \sim T^{-2}$  in a nuclear matter in NS [19]. The instability of the magnetic field in Eq. (5) proceeds from the terms containing  $\mu_{5q}$  and  $V_{5q}$ . Thus, to avoid the excessive growth of the magnetic field, it is sufficient to replace  $T^2 \rightarrow T^2 (1 + B^2/B_{\text{eq}}^2)^{-1}$  or  $\Pi \rightarrow \Pi [1 + B^2/B_{\text{eq}}^2(T)]^{-1}$  only in these terms [9].
- [22] E. Braaten, Neutrino emissivity of an ultrarelativistic plasma from positron and plasmino annihilation, Astrophys. J. 392 (1992) 70–73.
- [23] V. A. Rubakov, On the electroweak theory at high fermion density, Prog. Theor. Phys. 75 (1986) 366–385.
- [24] V. Dexheimer, S. Schramm, A Novel approach to model hybrid stars, Phys. Rev. C 81 (2010) 045201, arXiv:0901.1748.
- [25] J. Kapusta, T. Toimela, Friedel oscillations in relativistic QED and QCD, Phys. Rev. D 37 (1988) 3731–3736.

- [26] V. B. Berestetskii, E. M. Lifschitz, L. P. Pitaevskii, *Quantum Electrodynamics*, 2nd ed., Pergamon, Oxford, 1982.
- [27] E. Braaten, D. Segel, Neutrino energy loss from the plasma process at all temperatures and densities, Phys. Rev. D 48 (1993) 1478–1491, hep-ph/9302213.